Homogeneous Linear Differential Equations Differential Equations X. Du

- Second-Order Linear Homogeneous Differential Equation
  - Take the form $y''+P(x)y'+Q(x)y=0$
  - General solution takes the form $y=c_1y_1+c_2y_2$
  - How to solve:
    - Find two distinct, linearly independent solutions (i.e. y_1 and y_2 are not linear multiples of each other – there exist no constants c_1 and c_2 such that $c_1y_1=c_2y_2$).
    - Apply superposition: $y=c_1y_1+c_2y_2$
    - If $P(x)$ and $Q(x)$ are constant, then use characteristic equation method (next page).
    - Apply these same principles to higher order linear homogeneous differential equations
  - An IVP involving a second-order differential equation requires two initial conditions (i.e. $y(x_o)=A$ and $y'(x_o)=B$).
    - To solve this IVP, substitute in the initial conditions and solve for c_1 and c_2
  - Existence/Uniqueness: Let $P(x)$ and $Q(x)$ be continuous on an interval $I\subseteq\mathbb{R}$. Then for some $x_o\in I$, the IVP $y''+P(x)y'+Q(x)y=0$, $y(x_o)=A$, $y'(x_o)=B$ has exactly one solution.
    - Can be used to prove that the general solution is $y=c_1y_1+c_2y_2$.
- An *n*th-order homogeneous linear differential equation takes the form $y^{(n)}+A(x)y^{(n-1)}+...+P(x)y'+Q(x)y=0$.
  - Has exactly *n* linearly independent solutions, each of which is not a superposition of other solutions
  - General solution takes the form $y=c_1y_1+c_2y_2+...+c_ny_n$ (application of superposition)
  - How to solve:
    - Find *n* distinct, linearly independent solutions, each of which is not a superposition of other solutions
    - Apply superposition: $y=c_1y_1+c_2y_2+...+c_ny_n$
    - If the coefficients of the differential equation are constant, then use a characteristic equation. This process is the same as for second-order equations, as shown on the next page.
  - An IVP involving an *n*th-order differential equation requires *n* initial conditions e.g. $y^{(n)}(0)=A$, $y^{(n-1)}(0)=B$,…, $y(0)=y_o$.
    - To solve this IVP, just substitute in the initial conditions and solve for the constants using a systems of equations.